

# Wormhole inspired by non-commutative geometry

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## Abstract

In the present letter we search for a new wormhole solution inspired by noncommutative geometry with the additional condition of allowing conformal Killing vectors (CKV). A special aspect of noncommutative geometry is that it replaces point-like structures of gravitational sources with smeared objects under Gaussian distribution. However, the purpose of this paper is to obtain wormhole solutions with non-commutative geometry as a background where we consider a point-like structure of gravitational object without smearing effect. It is found through this investigation that wormhole solutions exist in this Lorentzian distribution with viable physical properties.

*Key words:* General Relativity; noncommutative geometry; wormholes

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## 1 INTRODUCTION

A *wormhole*, which is similar to a tunnel with two ends each in separate points in spacetime or two connecting black holes, was conjectured first by Weyl [1]

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and later on by [2]. In a more concrete physical definition it is essentially some kind of hypothetical topological feature of spacetime which may acts as *shortcut* through spacetime topology.

It is argued by Morris et al. [3,4] and others [5,6,7] that in principle a wormhole would allow travel in time as well as in space and can be shown explicitly how to convert a wormhole traversing space into one traversing time. However, there are other types of wormholes available in the literature where the traversing path does not pass through a region of exotic matter [8,9]. Following the work of Visser [8], a new type of thin-shell wormhole, which was constructed by applying the cut-and-paste technique to two copies of a charged black hole [10], is of special mention in this regard.

Thus a traversable wormhole, tunnel-like structure connecting different regions of our Universe or of different universes altogether, has been an issue of special investigation under Einstein's general theory of relativity [11]. It is argued by Rahaman et al. [12] that although just as good a prediction of Einstein's theory as black holes, wormholes have so far eluded detection. As one of the peculiar features a wormhole requires the violation of the null energy condition (NEC) [4]. One can note that phantom dark energy also violates the NEC and hence could have deep connection to in formation of wormholes [13,14].

It is believed that some perspective of quantum gravity can be explored mathematically in a better way with the help of non-commutative geometry. This is based on the non-commutativity of the coordinates encoded in the commutator,  $[x_\mu, x_\nu] = \theta_{\mu\nu}$ , where  $\theta_{\mu\nu}$  is an anti-symmetric and real second-ordered matrix which determines the fundamental cell discretization of spacetime [15,16,17,18,19]. We also invoke the inheritance symmetry of the spacetime under conformal Killing vectors (CKV). Basically CKVs are motions along which the metric tensor of a spacetime remains invariant up to a certain scale factor. In a given spacetime manifold  $M$ , one can define a globally smooth conformal vector field  $\xi$ , such that for the metric  $g_{ab}$  it can be written as

$$\xi_{a;b} = \psi g_{ab} + F_{ab}, \quad (1)$$

where  $\psi : M \rightarrow R$  is the smooth conformal function of  $\xi$  and  $F_{ab}$  is the conformal bivector of  $\xi$ . This is equivalent to the following form:

$$L_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik}, \quad (2)$$

where  $L$  signifies the Lie derivatives along the CKV  $\xi^\alpha$ .

In favor of the prescription of this mathematical technique CKV we find out the following features: (1) it provides a deeper insight into the spacetime geometry and facilitates the generation of exact solutions to the Einstein field

equations in a more comprehensive forms, (2) the study of this particular symmetry in spacetime is physically very important as it plays a crucial role of discovering conservation laws and to devise spacetime classification schemes, and (3) because of the highly non-linearity of the Einstein field equations one can reduce easily the partial differential equations to ordinary differential equations by using CKV. Interested readers may look at the recent works on CKV technique available in the literature [20,21,22].

In this letter therefore we search for some new solutions of wormhole admitting conformal motion of Killing Vectors. It is a formal practice to consider the inheritance symmetry to establish a natural relationship between spacetime geometry and matter-energy distribution for a astrophysical system. Thus our main goal here is to examine the solutions of Einstein field equations by admitting CKV under non-commutative geometry. The scheme of the investigation is as follows: in the Sec. 2 we provide the mathematical formalism and Einstein's field equations under the framework of this technique. A specific matter-energy density profile has been employed in Sec. 3 to obtain various physical features of the wormhole under consideration by addressing the issues like the conservation equation, stability of the system, active gravitational mass and gravitational energy. Sec. 4 is devoted for some concluding remarks.

## 2 CONFORMAL KILLING VECTOR AND BASIC EQUATIONS

We take the static spherically symmetric metric in the following form

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

where  $r$  is the radial coordinate. Here  $\nu$  and  $\lambda$  are the metric potentials which have functional dependence on  $r$  only.

Thus, the only survived Einstein's field equations in their explicit forms (rendering  $G = c = 1$ ) are

$$e^{-\lambda} \left[ \frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} = 8\pi\rho, \quad (4)$$

$$e^{-\lambda} \left[ \frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = 8\pi p_r, \quad (5)$$

$$\frac{1}{2}e^{-\lambda} \left[ \frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda') \right] = 8\pi p_t, \quad (6)$$

where  $\rho$ ,  $p_r$  and  $p_t$  are matter-energy density, radial pressure and transverse pressure respectively for the fluid distribution. Here  $\prime$  over  $\nu$  and  $\lambda$  denotes partial derivative w.r.t. radial coordinate  $r$ .

The conformal Killing equations, as mentioned in Eqs. (2), then yield as follows:

$$\xi^1 \nu' = \psi,$$

$$\xi^4 = C_1 = \text{constant},$$

$$\xi^1 = \frac{\psi r}{2},$$

$$\xi^1 \lambda' + 2\xi_{,1}^1 = \psi,$$

where  $\xi^\alpha$  are the conformal 4-vectors and  $\psi$  is the conformal function as mentioned earlier.

This set of equations, in a straight forward way, imply the following simple forms:

$$e^\nu = C_2^2 r^2, \tag{7}$$

$$e^\lambda = \frac{C_3^2}{\psi^2}, \tag{8}$$

$$\xi^i = C_1 \delta_4^i + \left( \frac{\psi r}{2} \right) \delta_1^i, \tag{9}$$

where  $C_2$  and  $C_3$  are integration constants. Here the non-zero components of the conformal Killing vector  $\xi^a$  are  $\xi^0$  and  $\xi^1$ .

Now using solutions (7) and (8), the equations (3)-(5) take the following form as

$$\frac{1}{r^2} \left[ 1 - \frac{\psi^2}{C_3^2} \right] - \frac{2\psi\psi'}{C_3^2 r} = 8\pi\rho, \tag{10}$$

$$\frac{1}{r^2} \left[ 1 - \frac{3\psi^2}{C_3^2} \right] = -8\pi p_r, \tag{11}$$

$$\left[ \frac{\psi^2}{C_3^2 r^2} \right] + \frac{2\psi\psi'}{C_3^2 r} = 8\pi p_t. \tag{12}$$

These are the equations forming master set which has all the information of the fluid distribution under the framework of Einstein's general theory of relativity with the associated non-commutative geometry and conformal Killing vectors.

### 3 THE MATTER-ENERGY DENSITY PROFILE AND PHYSICAL FEATURES OF THE WORMHOLE

As stated by Rahaman et al. [11], the necessary ingredients that supply fuel to construct wormholes remain an elusive goal for theoretical physicists and there are several proposals that have been put forward by different authors [23,24,25,26,27,28]. However, in our present work we consider cosmic fluid as source and thus have provided a new class of wormhole solutions. Keeping the essential aspects of the noncommutativity approach which are specifically sensitive to the Gaussian nature of the smearing as employed by Nicolini et al. [18], we rather get inspired by the work of Mehdipour [29] to search for a new fluid model admitting conformal motion. Therefore, we assume a Lorentzian distribution of particle-like gravitational source and hence the energy density profile as given in Ref. [29] as follows:

$$\rho(r) = \frac{M\sqrt{\phi}}{\pi^2(r^2 + \phi)^2}, \quad (13)$$

where  $\phi$  is the noncommutativity parameter and  $M$  is the smeared mass distribution.

Now, solving equation (10) we get

$$\psi^2 = C_3^2 - \left( \frac{4C_3^2 M}{\pi r} \right) \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right] + \frac{D_1}{r}, \quad (14)$$

where  $D_1$  is an integration constant and can be taken as zero.

The parameters, like the radial pressure, tangential pressure and metric potentials, are found as

$$p_r = \frac{1}{8\pi} \left[ \frac{2}{r^2} - \frac{12M}{\pi r^3} \left\{ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right\} \right], \quad (15)$$

$$p_t = \frac{1}{8\pi} \left[ \frac{1}{r^2} - 8\pi \left( \frac{M\sqrt{\phi}}{\pi^2(r^2 + \phi)^2} \right) \right], \quad (16)$$

$$e^\nu = C_2^2 r^2, \quad (17)$$

$$e^\lambda = \frac{1}{\left[ 1 - \left( \frac{4M}{\pi r} \right) \left( \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right) \right]}. \quad (18)$$

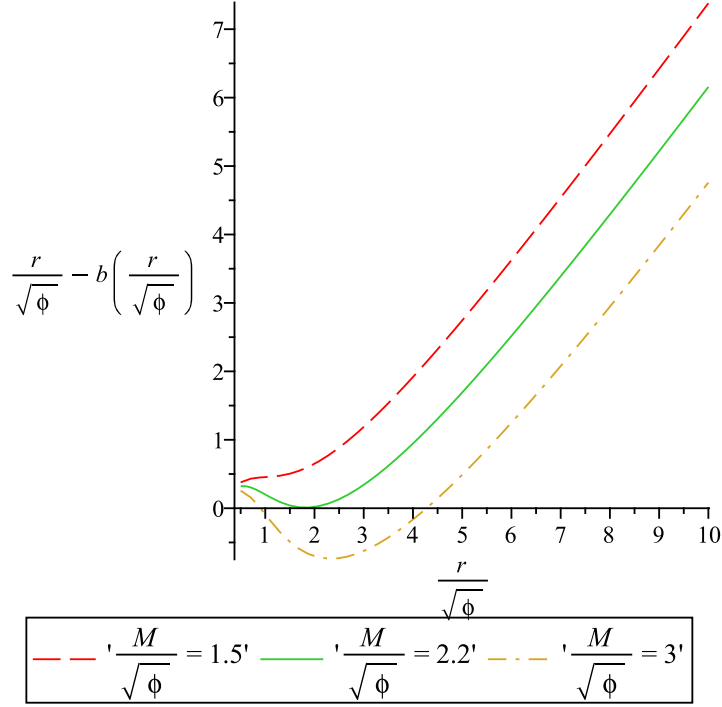


Fig. 1. The throat of the wormhole is located at  $\frac{r}{\sqrt{\phi}} = \frac{r_0}{\sqrt{\phi}}$  (maximum root), where  $b\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r}{\sqrt{\phi}}$  cuts the  $\frac{r}{\sqrt{\phi}}$ -axis. For  $\frac{M}{\sqrt{\phi}} < 2.2$ , there exists no root, and therefore no throats. For  $\frac{M}{\sqrt{\phi}} > 2.2$ , however we have two roots: (i) for  $\frac{M}{\sqrt{\phi}} = 3$ , the location of the external root i.e. throat of the wormhole is  $\frac{r_0}{\sqrt{\phi}} = 4.275$ , and (ii) for  $\frac{M}{\sqrt{\phi}} = 2.2$ , we have one and only one solution and this corresponds to the situation when two roots coincide and it can be interpreted as an extreme situation

Let us now write down the metric potential conveniently in terms of the shape function  $b(r)$  as follows:

$$e^\lambda = \frac{1}{1 - \frac{b(r)}{r}}, \quad (19)$$

where  $b(r)$  is given by

$$b(r) = \left(\frac{4M}{\pi}\right) \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right]. \quad (20)$$

Now, we will discuss the behavioral effects of different aspects of the above shape function  $b(r)$  and its derivative. The throat location of the wormhole is obtained by imposing the equation  $b(r_0) = r_0$ . One can note that the appearance of a throat depends on the parameter  $M$  and  $\phi$ . However, the larger root

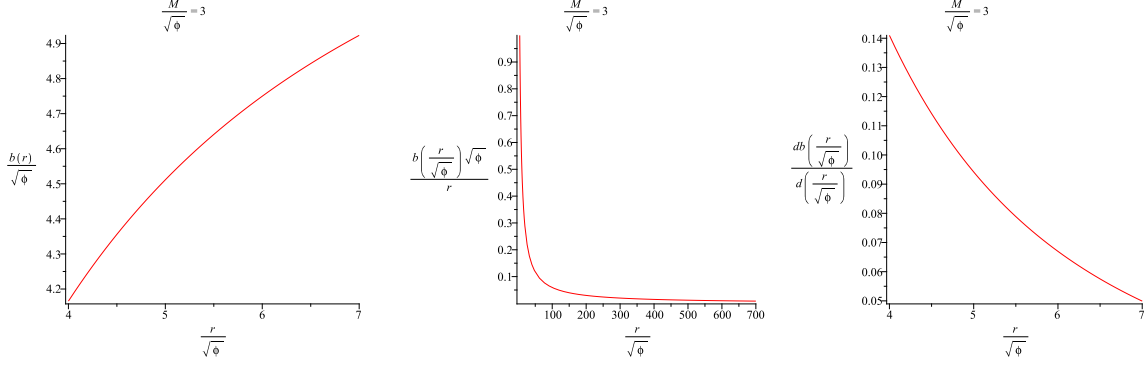


Fig. 2. (Left) Diagram of the shape function of the wormhole for the specific value of the parameter  $\frac{M}{\sqrt{\phi}} = 3$ . (Middle) Diagram of the asymptotic behaviour of shape function. (Right) Diagram of the derivative of the shape function of the wormhole.

of the equation  $b\left(\frac{r_0}{\sqrt{\phi}}\right) = \frac{r_0}{\sqrt{\phi}}$ , where  $\frac{r_0}{\sqrt{\phi}}$  is dimensionless, gives the throat which depends only one parameter  $\frac{M}{\sqrt{\phi}}$ . Figure 1 shows that the throat of the wormhole is located at  $\frac{r}{\sqrt{\phi}} = \frac{r_0}{\sqrt{\phi}}$  (maximum root), where  $\frac{r}{\sqrt{\phi}} - b\left(\frac{r_0}{\sqrt{\phi}}\right)$  cuts the  $\frac{r}{\sqrt{\phi}}$ -axis. One can note that position of the throat is increasing with the increase of smeared mass distribution  $M$ . For  $\frac{M}{\sqrt{\phi}} < 2.2$  no throat exists. From the above analysis, we notice that we may get feasible wormholes for  $\frac{M}{\sqrt{\phi}} > 2.2$ . For the sake of brevity, we assume  $\frac{M}{\sqrt{\phi}} = 3$  for the rest of the study.

From the left panel of Fig. 2, we observe that shape function is increasing, therefore,  $b'\left(\frac{r}{\sqrt{\phi}}\right) > 0$ . From Fig. 1, one can also note that for  $\left(\frac{r}{\sqrt{\phi}}\right) > \left(\frac{r_0}{\sqrt{\phi}}\right)$ ,  $\left(\frac{r}{\sqrt{\phi}}\right) - b\left(\frac{r_0}{\sqrt{\phi}}\right) > 0$ . This immediately implies that  $\frac{b\left(\frac{r}{\sqrt{\phi}}\right)}{\left(\frac{r}{\sqrt{\phi}}\right)} < 1$  which is an essential requirement for a shape function. Right panel of figure 2 indicates that the flare-out condition  $b'\left(\frac{r}{\sqrt{\phi}}\right) < 1$  for  $\left(\frac{r}{\sqrt{\phi}}\right) > \left(\frac{r_0}{\sqrt{\phi}}\right)$  is satisfied.

We also observe the asymptotic behaviour from the middle panel of Fig. 2 such that  $\frac{b\left(\frac{r}{\sqrt{\phi}}\right)}{\left(\frac{r}{\sqrt{\phi}}\right)} \rightarrow 0$  as  $\left(\frac{r}{\sqrt{\phi}}\right) \rightarrow \infty$ . Unfortunately, this has the similar explanation as done in Ref. [20] that the redshift function does not approach zero as  $r \rightarrow \infty$  due to the conformal symmetry. This means the wormhole spacetime is not asymptotically flat and hence will have to be cut off at some radial distance which smoothly joins to an exterior vacuum solution.

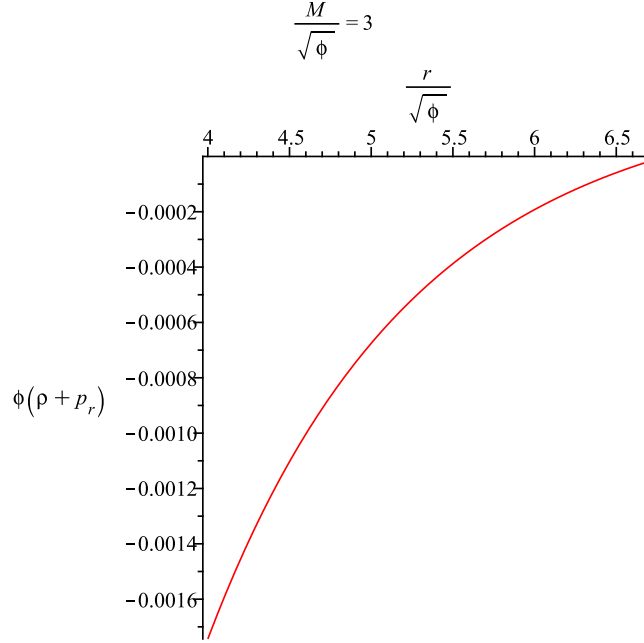


Fig. 3. The violation of null energy condition is shown against  $\left(\frac{r}{\sqrt{\phi}}\right)$ .

One can now find out the redshift function  $f(r)$ , where  $e^{2f(r)} = e^{\nu(r)}$ . Using Eq. (7) we find

$$f(r) = \ln(C_2 r). \quad (21)$$

It can be observed from the above expression that the wormhole presented here is traversable one as redshift function remains finite.

The above solution should be matched with the exterior vacuum spacetime of the Schwarzschild type at some junction interface with radius  $R$ . Using this matching condition, one can easily find the value of unknown constant  $C_2$  as

$$C_2 = \frac{e^{f(R)}}{R}, \quad (22)$$

so that the redshift function now explicitly becomes

$$f(r) = \ln \left[ \frac{r e^{f(R)}}{R} \right]. \quad (23)$$

The redshift function is therefore finite in the region  $r_0 < r < R$ , as required because this will prevent an event horizon. According to Fig. 3,  $\phi(\rho + p_r) < 0$ , therefore, the null energy condition is violated to hold a wormhole open.



### 3.1 THE TOLMAN-OPPENHEIMER-VOLKOFF EQUATION

Following the suggestion of Ponce de Leon [30], we write the Tolman-Oppenheimer-Volkoff (TOV) equation in the following form

$$-\frac{M_G(\rho + p_r)}{r^2}e^{\frac{\lambda-\nu}{2}} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0, \quad (24)$$

where  $M_G = M_G(r)$  is the effective gravitational mass within the region from  $r_0$  up to the radius  $r$  and is given by

$$M_G(r) = \frac{1}{2}r^2 e^{\frac{\nu-\lambda}{2}} \nu'. \quad (25)$$

Equation (24) expresses the equilibrium condition for matter distribution comprising the wormhole subject to the gravitational force  $F_g$ , hydrostatic force  $F_h$  plus another force  $F_a$  due to anisotropic pressure. Now, the above Eq. (24) can be easily written as

$$F_g + F_h + F_a = 0, \quad (26)$$

where

$$F_g = -\frac{\nu'}{2}(\rho + p_r) = -\frac{1}{4\pi r^3} - \frac{M\sqrt{\phi}}{\pi^2 r(r^2 + \phi)^2} - \frac{3M\sqrt{\phi}}{2\pi^2 r^3(r^2 + \phi)} + \frac{3M}{2\pi^2 r^4} \tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right), \quad (27)$$

$$F_h = -\frac{dp_r}{dr} = \frac{1}{2\pi r^3} + \frac{3M\sqrt{\phi}}{\pi^2 r^3(r^2 + \phi)} + \frac{3M\sqrt{\phi}}{\pi^2 r(r^2 + \phi)^2} - \frac{9M}{2\pi^2 r^4} \tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) + \frac{3M}{2\pi^2 r^3(r^2 + \phi)}, \quad (28)$$

$$F_a = \frac{2}{r}(p_t - p_r) = -\frac{1}{4\pi r^3} - \frac{2M\sqrt{\phi}}{\pi^2 r(r^2 + \phi)^2} - \frac{3M\sqrt{\phi}}{\pi^2 r^3(r^2 + \phi)} + \frac{3M}{\pi^2 r^4} \tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right). \quad (29)$$

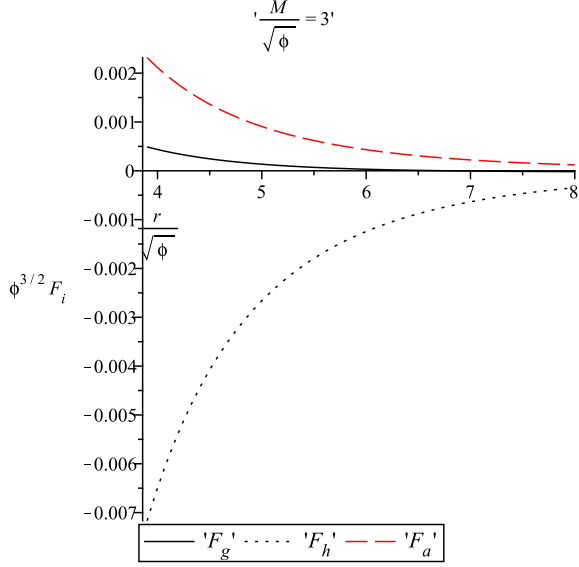


Fig. 4. The variation of  $\phi^{(3/2)} \times$  forces are shown against  $\frac{r}{\sqrt{\phi}}$ .

From the Fig. 4 it can be observed that stability of the system has been attained by gravitational and anisotropic forces against hydrostatic force.

### 3.2 ACTIVE GRAVITATIONAL MASS

The active gravitational mass within the region from the throat  $r_0$  up to the radius  $R$  can be found as

$$M_{active} = 4\pi \int_{r_0+}^R \rho r^2 dr = \frac{2M}{\pi} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right]_{r_0+}^R. \quad (30)$$

We observe here that the active gravitational mass  $M_{active}$  of the wormhole is positive under the constraint  $\tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) > \frac{r\sqrt{\phi}}{r^2 + \phi}$  and also the nature of variation is physically acceptable as can be seen from Fig. 5.

### 3.3 TOTAL GRAVITATIONAL ENERGY

Using the prescription given by Lyndell-Bell et al. [31] and Nandi et al. [32], we calculate the total gravitational energy of the wormhole as

$$E_g = Mc^2 - E_M, \quad (31)$$

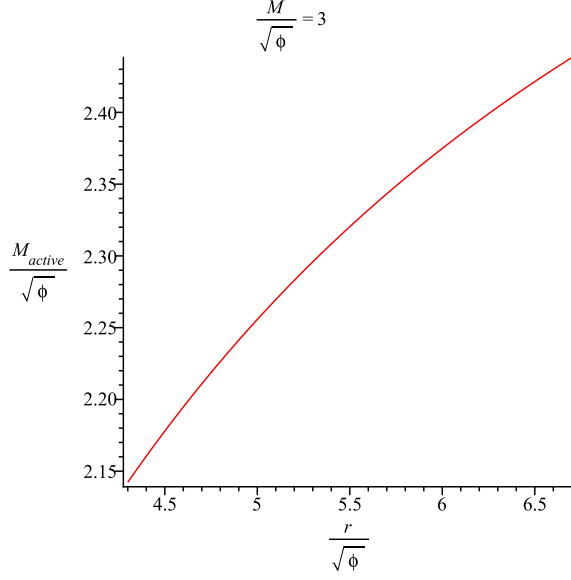


Fig. 5. The variation of  $\frac{M_{active}}{\sqrt{\phi}}$  is shown against  $\frac{r}{\sqrt{\phi}}$ .

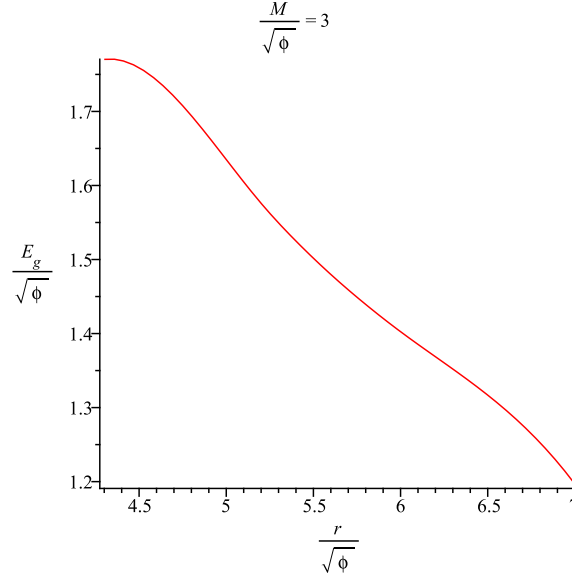


Fig. 6. The variation of  $\frac{E_g}{\sqrt{\phi}}$  is shown against  $\frac{r}{\sqrt{\phi}}$ . Fig. 1 indicates that for  $\frac{M}{\sqrt{\phi}} = 3$ ,  $\frac{r_0}{\sqrt{\phi}}$  takes the value 4.275.

where,  $Mc^2 = \frac{1}{2} \int_{r_0}^R T_0^0 r^2 dr + \frac{r_0}{2}$  is the total energy and  $E_M = \frac{1}{2} \int_{r_0}^R \sqrt{g_{rr}} \rho r^2 dr$  is the total mechanical energy. Note that here  $\frac{4\pi}{8\pi}$  yields the factor  $\frac{1}{2}$ .

The range of the integration is considered here from the throat  $\frac{r_0}{\sqrt{\phi}}$  to the embedded radial space of the wormhole geometry. We have solved the above Eq. (31) numerically.

Table 1  
Data for plotting Fig. 6

Upper limit $R$	Value of $E_g$
5.0	1.635059734
5.2	1.575910504
5.4	1.524883013
5.6	1.479809791
5.8	1.439327656
6.0	1.402514488
6.2	1.368712897

In Fig. 6 we have considered  $\frac{M}{\sqrt{\phi}} = 3$ , throat radius  $\frac{r_0}{\sqrt{\phi}} = 4.275$ , the upper limit  $\frac{R}{\sqrt{\phi}}$  is varying from 4.275+ to 7. We have prepared a data sheet in Table 1 for plotting Fig. 6. Here our observations are as follows: (1) When we are taking  $\frac{r_0}{\sqrt{\phi}}$  less than 4.275, the value of the integration become complex; (2) The numerical value of the integration becomes real from the range of lower limit 4.275+. These real and positive values imply  $E_g > 0$ , which at once indicates that there is a repulsion around the throat. Obviously this result is expected for construction of a physically valid wormhole to maintain stability of the fluid distribution.

#### 4 CONCLUDING REMARKS

In the present letter we have considered anisotropic real matter source for constructing new wormhole solutions. The background geometry is inspired by noncommutativity along with conformal Killing vectors to constrain the form of the metric tensor. Speciality of this noncommutative geometry is to replace point-like structure of gravitational source by smeared distribution of the energy density under Gaussian distribution. Notably, in this work we consider a point-like structure of gravitational object without smearing effect where matter-energy density is of the form provided by Mehdipour [29].

Our investigation indicates that traversable wormhole solutions exist in this Lorentzian distribution with physically interesting properties under appropriate conditions.

The main observational highlights of the present study therefore are as follows:

- (1) The stability of the matter distribution comprising of the wormhole has been attained in the present model. For this we have calculated the TOV equation which expresses the equilibrium condition for matter distribution subject to the gravitational force  $F_g$ , hydrostatic force  $F_h$  plus another force  $F_a$  due to anisotropic pressure.
- (2) The active gravitational mass  $M_{active}$  of the wormhole is positive under the constraint  $\tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) > \frac{r\sqrt{\phi}}{r^2 + \phi}$  as is expected from the physical point of view.
- (3) Since the total gravitational energy,  $E_g > 0$ , there is a repulsion around the throat which is expected usually for stable configuration of a wormhole.

As stated earlier in the text, in the present letter we employ energy density given by Mehdipour [29] instead of Nicolini-Smailagic-Spallucci type [18]. However, our overall observation is that in our present approach the solutions and properties of the model are physically valid and interesting as much as in the former approach. As a special mention we would like to look at the Fig. 2 where we observe that the shape function is increasing instead of monotone increase as in the former case (Fig. 2 of Ref. [20]). Further, the redshift function does not approach zero as  $r > r_0$  due to the conformal symmetry in both the approaches. So, exploration can be done with some other rigorous studies between the two approaches, i.e. Refs. [18] and [29], which can be sought for in a future project.

## Acknowledgments

F.R. and S.R. are thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), India for providing Visiting Research Associateship under which a part of this work was carried out. IK is also thankful to IUCAA for research facilities. F.R. is grateful to UGC, India for financial support under its Research Award Scheme (Reference No.: F.30-43/2011 (SA-II)). We are very grateful to an anonymous referee for his/her insightful comments that have led to significant improvements, particularly on the interpretational aspects.

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